Rank of Matrix

A number 'r' is said to be the rank of a matrix A, if it possesses the following two properties.

(i.) The determinant of atleast one minor of order 'r' is non-zero.

(ii.) The determinant of every minor of A of order higher than 'r' is zero.

Rank of a matrix A is denoted by $\rho(A)$

Properties of Rank

1.) The rank of every non-singular matrix of order n is n.

2.) The rank of a square matrix A of order n can be less than n if and only if A is singular i.e. $|A| = 0$

(iii.) The rank 'r' of an $m \times n$ matrix can atmost be equal to the smaller of the numbers m and n, but it may be less.

(iv.) The rank of zero matrix is taken as zero.

(v.) If I is a unit matrix of order n, then its rank is equal to n.

(vi.) Elementary transformation do not alter the rank of a matrix.

vii.) If A is a square matrix of order m then m-rank A is called the nullity of the matrix A and is denoted by N(A)

VIII. Rank of a matrix is same as the number of linearly independent row vectors in the matrix as well as the number of L.I column vectors in the matrix.

(IX.) Rank of a Matrix A does not change by pre multiplication or post-multiplication with any non-singular Matrix.

X. If A and B are matrices of same order, then $\rho(A+B) \leq (\rho(A)+\rho(B))$

XI If A and B are matrices of same order, then

$\rho(AB) \leq \min(\rho(A), \rho(B))$

where

A and B are conformable for multiplication.

Thus $\rho(AB) \leq \rho(A)$

and $\rho(AB) \leq \rho(B)$

XII. If A' is a transpose of matrix A then

$\rho(A') = \rho(A)$ and $\rho(AA') = \rho(A)$

XIII. If A^θ is the Conjugate transpose of A, then $\rho(A^\theta) = \rho(A)$ and

$\rho(AA^\theta) = \rho(A)$